

Important Concepts for Stat 61

Calculus and Algebra Exercises:

1. Find the derivative with respect to x of $f(x) = xe^{-x}$.
2. Find the derivative with respect to x of $f(x) = a^x$ (assume $a > 0$).
3. Find the antiderivative of $f(x) = \log(x)$ (i.e., evaluate the indefinite integral $\int \log(x)dx$).
4. Find the antiderivative of $f(x) = xe^{-x}$.
5. Show that $\int_{-\infty}^{\infty} e^{-x^2/2} dx = \sqrt{2\pi}$. Hint: Show that the square of the integral equals 2π . Write it as a double integral for variables x and y and convert to polar coordinates.
6. Show that $\sum_{n=0}^{\infty} \frac{a^n}{n!} = e^a$ (Hint: it's a Taylor series expansion about $a = 0$).
7. Show that $\lim_{n \rightarrow \infty} (1 + a/n)^n = e^a$ (Hint: use the binomial expansion).
8. For x a fixed, positive integer, show that $\lim_{n \rightarrow \infty} \frac{n!/(n-x)!}{n^x} = 1$.
9. Show that, for $a > 0$,

$$\int_0^{\infty} x^a e^{-x} dx = a \int_0^{\infty} x^{a-1} e^{-x} dx.$$

10. Find the value of μ (in terms of the x_i 's) that maximizes the function

$$L(\mu) = \exp \left[-\frac{1}{2} \sum_{i=1}^n (x_i - \mu)^2 / 2 \right].$$

11. Show that

$$\sum_{i=1}^n (x_i - \mu)^2 = n(\bar{x} - \mu)^2 + \sum_{i=1}^n (x_i - \bar{x})^2, \quad \text{where } \bar{x} = \frac{1}{n} \sum_{i=1}^n x_i.$$

12. Suppose $f(x, y) = 4e^{-2y}$ for $0 < x < y$, and $f(x, y) = 0$ otherwise. Show that the total volume under f is 1. Also find the volume over the region where $x > 1$.

Elementary Statistics Concepts:

Elementary statistics (e.g., AP Stat, Stat 11 or Econ 31) is not required for Stat 61, but it is "strongly recommended".

1. What is the difference between an **experiment** and an **observational study**, and why is the distinction important?
2. What is the difference between a **Simple Random Sample** (SRS) and a **Stratified Random Sample**?
3. What is a **test of hypotheses**, and what is the definition of the **P-value** for a test?
4. What does it mean to say a result is **statistically significant**? What does it mean to say a result is **not statistically significant**?
5. What does it mean if the **margin of error** for a 95% **confidence interval** is 0.03?
6. What is meant by the **sampling distribution** for a **statistic**?